Section 6.5 - Direct Comparison Test and Limit Comparison Test (Examples)

Nov. 12, 2021

Friday

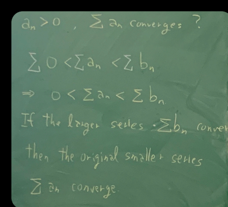
Direct Comparison Test

$\Sigma 0 < \Sigma a_n < \Sigma b_n$ (will be positive eventually)

apply to all

Direct Comparison Test CASE I

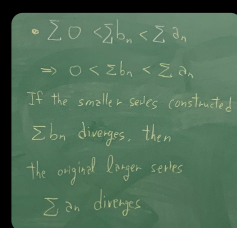
- $0 < \Sigma a_n < \Sigma b_n$ → if Σb_n Converges, then Σa_n, the original smaller series must converge as well



OR

Direct Comparison Test CASE II

- $0 < \Sigma b_n < \Sigma a_n$ → if Σb_n diverges then Σa_n, the original smaller series must diverge as well



6.5.1.D

$$\sum_{n=1}^{\infty} \frac{4n^2 - 3}{n^2 \cdot 5^n}$$

$4n^2 - 3$ ← 2 terms (terms are seperated by + or −)

$n^2 \cdot 5^n$ ← one term, 2 factors

* drop less significant *

$$0 < \sum \frac{4n^2-3}{n^2 \cdot 5^n} < \sum \frac{4\cancel{n^2}}{\cancel{n^2} \cdot 5^n}$$

Do sum starting from where it is positive, so if this was positive @ 2 start from 2 instead of 1

$$0 < \sum \frac{4n^2-3}{n^2 \cdot 5^n} < \sum \frac{4}{5^n}$$

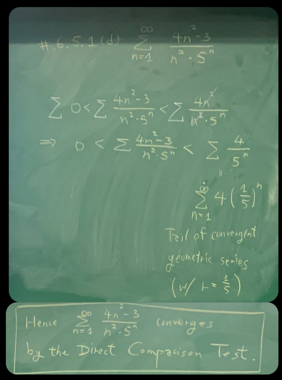
$$= \sum_{n=1}^{\infty} 4\left(\boxed{\tfrac{1}{5}}\right)^n$$

$\uparrow$ r

tail of convergent geometric series (since starts @ 1 instead of zero)

Hense we can conclude $\frac{4n^2-3}{n^2 \cdot 5^n}$ converges by the direct comparison test

ANSWER



6.5.1.A

$$\sum_{n=1}^{\infty} \frac{1}{3 * \sqrt[4]{n-1}} = \frac{1}{3} \sum_{n=2}^{\infty} \frac{1}{\sqrt[4]{n-1}}$$

If n=1 would be ∞ since zero in denominator

$$\sum 0 < \sum \frac{1}{3} \frac{1}{\sqrt[4]{n}} < \sum \frac{1}{3} \frac{1}{\sqrt[4]{n-1}}$$

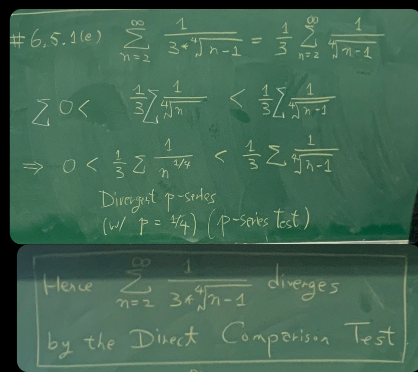
$$\sum 0 < \frac{1}{3}\sum \frac{1}{n^{1/4}} < \frac{1}{3}\sum \frac{1}{\sqrt[4]{n-1}}$$

* Divegant p series where p = 1/4, Column #4 *

Hense $\sum_{n=2}^{\infty} \frac{1}{3 \cdot \sqrt[4]{n-1}}$ diverges

by the direct Comparison test

ANSWER



6.5.1.A.

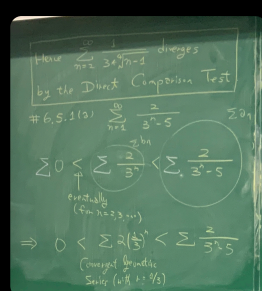
$$\sum_{n=1}^{\infty} \frac{2}{3^n - 5}$$

$$0 < \sum \frac{2}{3^n} < \sum \frac{2}{3^n - 5}$$

eventually (for n = 2, 3, ...)

$$\rightarrow 0 < \sum 2\left(\frac{1}{3}\right)^n < \sum \frac{2}{3^n - 5}$$

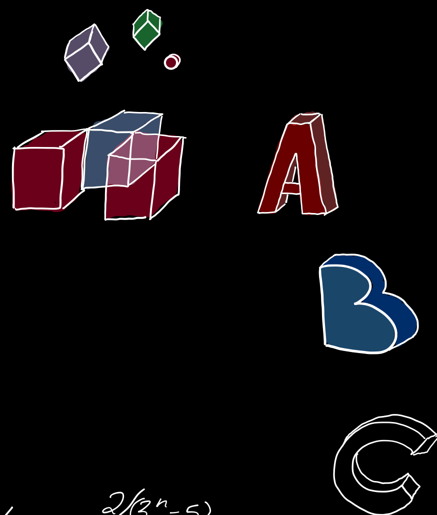
r ∴ Convergent geometric series



From direct Comparison Test **ANSWER**

Limit Comparison Test

Nov 15^{th}, 2021 Monday



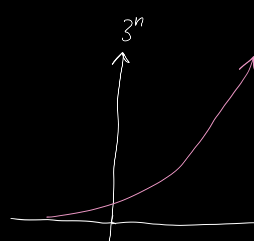
$$\sum_{n=1}^{\infty} \frac{2}{3^n-5}$$

$$0 < \sum \overset{b_n}{\frac{2}{3^n}} < \sum \overset{a_n}{\frac{2}{3^n-5}}$$

$$\lim_{n\to\infty} \frac{2/(3^n-5)}{2/(3^n)}$$

$$= \lim \frac{\cancel{2}}{3^n-5} * \frac{3^n}{\cancel{2}}$$

$$= \lim_{n\to\infty} \frac{3^n}{3^n-5} \quad \boxed{\frac{\infty}{\infty} \text{ Form}}$$



$$\overset{L'H}{=} \lim_{n\to\infty} \frac{\cancel{3^n} * \ln 3}{\cancel{3^n} * \ln 3}$$

$$\lim_{n\to\infty} 1 = 1 \ (=L)$$

$0 < 1 < \infty$

Since $\sum_{n=1}^{\infty} \frac{2}{3^n}$ Convergent

geometric series with $r = 1/3$; geometric series test

$\boxed{\text{Hense, } \sum_{n=1}^{\infty} \frac{2}{3^n-5} \text{ Converges by limit Comparison test}}$

6.5.1F

$$\sum_{n=1}^{\infty} \frac{n^2}{n^3 \boxed{+1}}$$

$$\Sigma\, 0 < \sum \frac{n^2}{n^3+1} < \sum \frac{n^2}{n^3}$$

$$0 < \sum_{n=1}^{\infty} \frac{n^2}{n^3+1} < \sum_{n=1}^{\infty} \boxed{\frac{1}{n}}$$

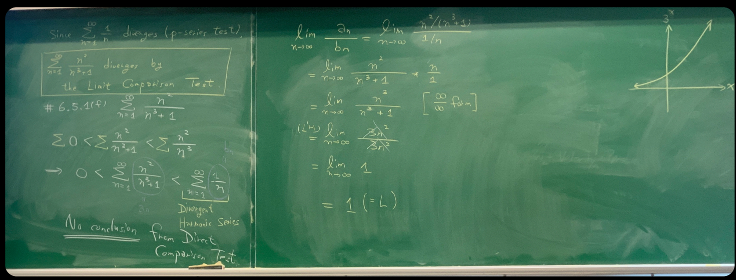
p series, p = 1, Collumn 4, harmonic series, diverges

No Conclusion from Direct Comparison Test

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim \frac{n^2/(n^3-1)}{1/n}$$

$$\lim_{n\to\infty} \frac{n^2}{n^3+1} \cdot \frac{n}{1}$$

$$= \lim_{n\to\infty} \frac{n^3}{n^3+1} = \frac{\infty}{\infty} \text{ form}$$



$$\stackrel{L'H}{=} \lim_{n\to\infty} \frac{3n^2}{3n^2}$$

$$\lim_{n\to\infty} 1 = 1\,(L)$$ if finite, and greater than zero, can apply limit comparison test

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (p-series test)

$\boxed{\sum_{n=1}^{\infty} \frac{n^2}{n^3+1} \text{ diverges by the limit Comparison Test}}$